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OPTIMUM PERFORMANCE IN AUDIO TRANSISTOR AMPLIFIERS USING RESISTIVE OR INDUCTIVE SOURCES

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This report describes the noise figure of a transistor amplifier in the grounded emitter configuration. The effects of biasing, and emitter resistances are included in the noise figure. When the signal source is an inductance such as the reproduce head of a magnetic re-recorder, the signal-to-noise ratio (S/N) is better measurement of amplifier performance. It is shown that the S/N ratio can be maximized by a suitable choice of inductance, depending upon the operating conditions of the transistor.

In amplifying very low signal levels, it is important to design the amplifier so that maximum signal amplification is achieved while minimum noise is introduced by the amplifier. Some noise is irreducible, so the problem of the designer is to minimize that noise which can be reduced. Apart from hum and background noises in a transistor amplifier, there are noise sources that can be minimized to provide the clearest output signal at a useable level.

NOISE SOURCES - THERMAL NOISE

Thermal noise is generated in any real resistance by the random motion of charged particles in the resistance. This motion gives rise to a mean squared voltage at the terminals of the resistance. This is computed by:

$$\overline{E_{nt}}^2 = 4KTRB$$

where $K = 1.28 \times 10^{-23}$ $\frac{\text{Watt - Sec.}}{\text{Degree}}$ (1)

R = Ohms

B = Bandwidth in cycles per second

T = Temperature in $^{\circ}$ K (approx. 290° at room temperature)

The thermal noise generated in a resistance can be represented by a noiseless resistor with a voltage generator of $\sqrt{4KTRB}$ in series with the resistor as shown in Figure (1).

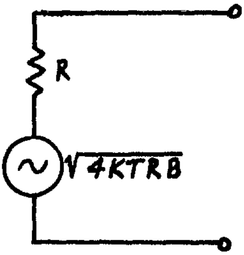

Figure 1

SHOT NOISE

Shot noise is developed whenever a current flows. It is caused by the discrete nature of the charged particles that travel through the device. The process, which like thermal noise is completely random, may be represented by a current generator I_{ns}, where:

$$\overline{I_{ns}^{2}} = 2qI_{d.c.}B \qquad (2)$$

where $q = 1.6 \times 10^{-19}$ coulombs

$I_{d.c.}$ = current flowing through the device

The equivalent circuit may be represented by a noiseless device shunted by a current generator of $\sqrt{2qI_{d.c.}B}$

1/f (FLICKER) NOISE

1/f or flicker noise (sometimes called "excess" noise) occurs in both semiconductors and tubes. In semiconductors it is generally considered to be the result of surface imperfections and transistor leakage. 1/f noise varies inversely with frequency, and gives rise to excess noise in transistors from approximately 1 KC down in frequency.

NOISE FIGURE

To evaluate the performance of an amplifier, the noise Figure (F), is taken as the ratio between the total output noise power from the amplifier, and the output noise power caused by the noise of the source. Thus, F is a measure of the increase in output noise power caused by the amplifier in relation to the output noise power introduced by the source.

The noise figure F can be taken over a band of frequencies or at a single frequency. (In the latter case, it is referred to as the spot noise figure.)

If any components are reactive or frequency sensitive, the numerator and denominator of the spot noise figure must be integrated with respect to frequency to determine the noise figure in the frequency band of interest.

OVERALL NOISE FIGURE OF SEVERAL AMPLIFIER STAGES

In a practical amplifier, in order to bring the signal up to a reasonable level, it is necessary to have several stages of amplification. Each of these stages creates additional noise in the output of the amplifier, as indicated by Frii's formula:

$$F_T = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + - - - - - - \qquad (3)$$

where

F_T = Total noise figure
F_1 = Noise figure of first stage
F_2 = Noise figure of second stage, etc.
G_1 = Power gain of first stage
G_2 = Power gain of second stage, etc.

Thus, in order to measure the noise performance of the first stage, it is necessary that the noise figures of the following stages be kept at a minimum, and that the first stage gain be as large as possible. Also, from formula (3), if the gain of the first stage is large and the noise figures of the following stages are reasonable, the noise figure F_T is essentially determined by the first stage.

TRANSISTOR NOISE FIGURE VERSUS FREQUENCY

Much has been learned about transistor noise figure versus frequency from experimental investigation and theoretical analysis. A typical curve is plotted in Figure 2.

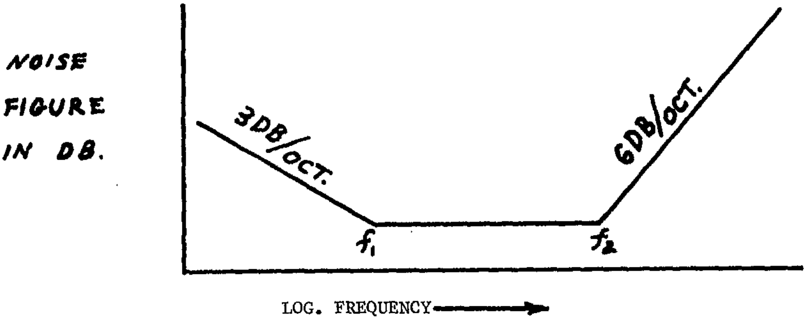

Figure 2

At frequencies below f_1, 1/f noise causes the noise figure to increase at approximately 3 db per octave, while above f_2 the noise figure increases approximately 6 db per octave because of the decrease in power gain of the transistor. In most transistors designed for low noise applications, the f_1 frequency is around 1 KC. f_2 is roughly equal to $\sqrt{f_\alpha \; f_\beta}$, the geometric mean of the common emitter and common base cutoff frequencies. It is important then, to select a transistor where the $\sqrt{f_\alpha \; f_\beta}$ frequency is above the upper bandwidth of the amplifier, or a transistor with the highest $\sqrt{f_\alpha \; f_\beta}$ frequency possible when the upper bandwidth of the amplifier extends into the area of transistor power gain loss.

TRANSISTOR NOISE EQUIVALENT CIRCUIT

Noise in transistors may be represented by appropriate generators placed in each leg of the "T", equivalent circuit as in Figure 3.

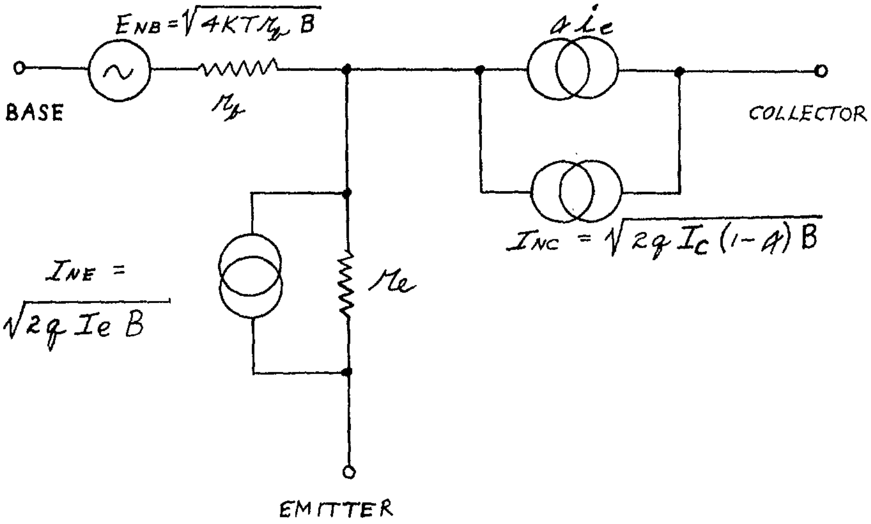

Figure 3

The emitter and collector noise generators are current generators, caused by the DC current flowing across these junctions. The base generator represents the thermal noise of the base resistance r_b. This transistor noise representation in Figure 3 follows Nielsen's [1] representation for transistor noise sources taken from van der Ziel[2]. The simplification over van der Ziel's model of noise sources in a transistor are:

1) The frequencies of interest are assumed to be below the cutoff frequencies of the transistor and the "T" equivalent impedances are not complex.

2) The collector and emitter noise generators are uncorrelated.

3) The collector leg impedance has been neglected. In small-signal transistors, r_c is generally of such magnitude that this introduces only a very small error.

4) 1/f noise is neglected.

For this representation, Nielsen has derived the noise figure of a transistor operating from a resistive source, and has obtained good agreement with experimental evidence. However, in most instances the transistor must be used in a circuit with additional elements; i.e., unbypassed emitter resistances, biasing resistors, and perhaps some form of feedback. For a transistor operating under these conditions, Halligan[3] has shown what increase in noise figure could be expected. Thus, in many cases the noise figure for the overall stage is a more realistic noise figure.

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[1] Nielsen, E.G., Behavior of Noise Figure in Junction Transistors, Proc. IRE Vol. 45, No. 7, p. 957, June 1957.

[2] van der Ziel, A., Shot Noise in Transistors, Proc. IRE Vol. 48, p. 114, Jan. 1960.

[3] Halligan, J. W., The Design of Low Noise Transistor Audio Amplifiers, Application Lab Report 687, Philco Corp., Lansdale Division.

Figure 4 represents a transistor with base and emitter resistors added and feedback generators placed in series with those resistors. Most transistor amplifiers can be put into this form by appropriate network theorems.

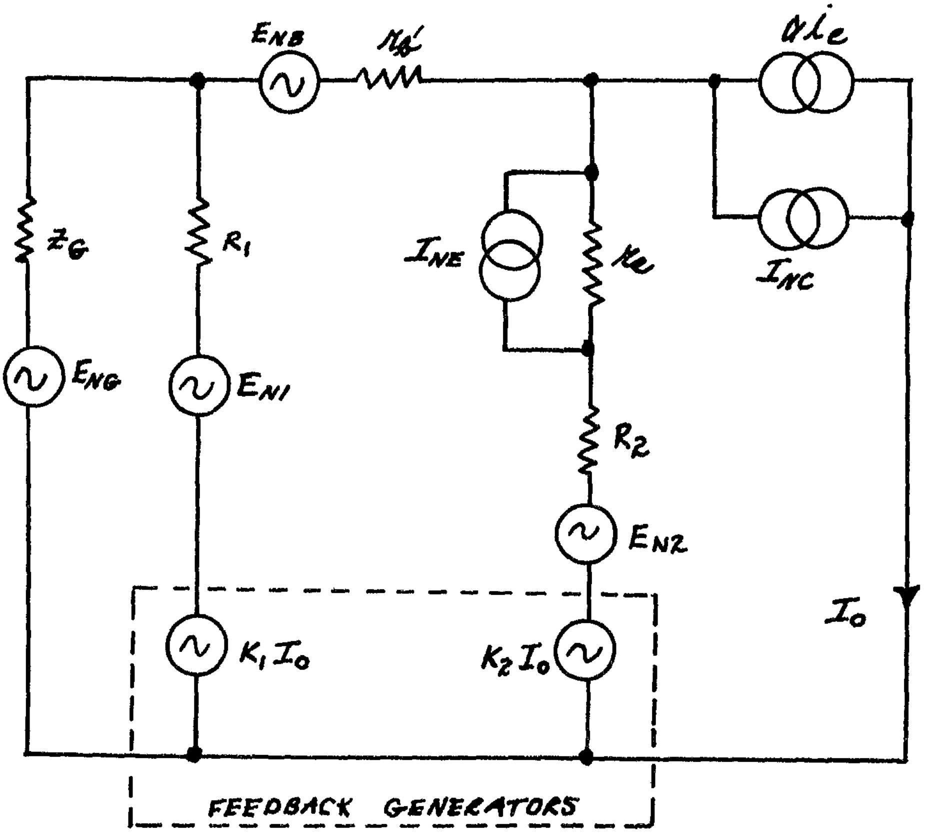

Figure 4

The sum of the output currents squared is found as:

$$I_o^2 = (E_{NG})^2 + (E_{N1})^2 \left|\frac{Z_G}{R_1}\right|^2 + (E_{NB})^2 \left|\frac{Z_G}{R_1} + 1\right|^2 + (E_{N2})^2 \left|\frac{Z_G}{R_1} + 1\right|^2 + (I_{NE})^2 r_e^2 \left|\frac{Z_G}{R_1} + 1\right|^2 + \left(\frac{I_{NC}}{\alpha}\right)^2 \left\{(r_e + r_b + R_2)\left|\frac{Z_G}{R_1} + 1\right| + |Z_G|\right\}^2 \tag{4}$$

where

$$(E_{NG})^2 = 4KTR_G B \tag{5}$$

$$(E_{NB})^2 = 4KT r_b B \tag{6}$$

$$(E_{N1})^2 = 4KTR_1 B \tag{7}$$

$$(E_{N2})^2 = 4KT R_2 B \tag{8}$$

$$(I_{NE})^2 = 2q I_e B \tag{9}$$

$$(I_{NC})^2 = 2q I_c (1-\alpha) B \tag{10}$$

TRANSISTOR CIRCUIT NOISE FIGURE

Since the noise figure is the ratio of the sum of the output currents squared to the output current caused by the source generator noise squared, the denominator of the output current is cancelled from the noise figure, and there is no need to compute it. It is important to note though, that the amplifier gain, input impedance, and effect of feedback is contained in the denominator, but that the noise figure does not depend upon it.

When the equivalent noise generators are placed in equation (4) and the following substitutions are made:

$$I_c \approx \alpha I_e \tag{11}$$

$$I_e = \frac{KT}{q r_e} \tag{12}$$

the noise figure of the amplifier becomes:

$$F = 1 + \frac{R_G}{R_1} + \frac{r_b}{R_G}\left(\frac{R_G}{R_1}+1\right)^2 + \frac{r_e}{2R_G}\left(\frac{R_G}{R_1}+1\right)^2 + \frac{R_2}{R_G}\left(\frac{R_G}{R_1}+1\right)^2 + \frac{\left[(r_e + r_b + R_2)\left(\frac{R_G}{R_1}+1\right) + R_G\right]^2}{2\beta r_e R_G} \qquad (13)$$

If R_1 (the base biasing resistor) in infinite and R_2 (the emitter resistor) is zero, the noise figure becomes:

$$F = 1 + \frac{r_b}{R_G} + \frac{r_e}{2R_G} + \frac{(r_e + r_b + R_G)^2}{2\beta r_e R_G} \qquad (14)$$

which is the equation derived by Nielsen.

In many cases, R_1 the base biasing resistor is very large compared to the source impedance R_G, and can be neglected in the noise figure calculations. The noise figure for the circuit then simplifies to:

$$F = 1 + \frac{r_b}{R_G} + \frac{r_e}{2R_G} + \frac{R_2}{R_G} + \frac{(r_e + r_b + R_G + R_2)^2}{2\beta r_e R_G} \qquad (15)$$

The increase of the noise figure by the bias resistor R_1 can be more severe as may be seen by equation (13). In order to keep the additional circuit noise at a minimum, R_1 should be much greater than R_G and R_2 should be much less than R_G.

OPTIMUM SOURCE RESISTANCE

The optimum source resistance (R_G opt.) is found by setting dF/dR_G to zero and solving for R_G opt. for the transistor only, without R_1 or R_2,

$$R_G \text{ opt.} = \sqrt{(\beta+1)2 r_e r_b + (\beta+1) r_e^2 + r_b^2} \qquad (16)$$

The optimum source resistance for a transistor with the emitter resistance R_2 in the circuit and a negligible value of bias resistor R_1 is:

$$\sqrt{(\beta+1)2 r_e r_b + (\beta+1) r_e^2 + r_b^2 + 2\beta r_e R_2 + R_2^2} \qquad (17)$$

Notice that R_G opt. has been increased by the addition of emitter resistor R_2. However, although R_G opt. has increased, the noise figure is still higher with R_2 than without it, when the optimum source resistance is used.

OPTIMUM re FOR GIVEN R_G

Setting dF/d re = 0 and assuming R_1 as very large compared to the source resistance, the optimum value is re is:

$$r_{e\,OPT} = \frac{r_b + R_G + R_2}{\sqrt{\beta + 1}} \tag{18}$$

The conclusion drawn from equation (18) is that re should be increased (I_c decreased) as the source resistance R_G is increased. There is a practical limit as to how far re can be increased (I_c decreased) and still maintain good stability and current gain (Beta). For example, if $R_G \gg$ than r_b and R_2 then:

$$r_{e\,OPT} \approx \frac{R_G}{\sqrt{\beta}} \tag{19}$$

At 100μa, re is approximately 270 ohms, therefore the maximum input source could only be $270\sqrt{\beta}$ ohms or about 2.6 K if β were 100. Thus, for high source resistances, operating into a grounded emitter transistor, optimum source matching for minimum noise figure is difficult if not impossible to attain. The best solution of course, is a matching input transformer if this device can be used. If not, a source resistance other than optimum may be used with some increase from the minimum noise figure.

NOISE WITH A REACTIVE SOURCE

If the source impedance is not resistive, but entirely reactive, the noise figure becomes meaningless - - since the source does not generate any noise. Magnetic tape recorders have as a signal source a reproduce head that is almost completely inductive. A measure of the noise qualities of such an amplifier is the signal-to-noise ratio, where the noise is the total noise power over the operating frequency band of the amplifier and the signal power is the equalized output from the amplifier.

Referring to Figure 4, Z_G is the reproduce head with an impedance of $2\pi f L_H$. E_{ng} would represent the open circuit voltage of the head, with the transistor and other circuit parameters remaining as shown. From equation (4) and equations (6) through (12), the signal power to noise power of an unequalized amplifier at any one frequency f is:

$$S/N = \frac{(E_H)^2/2KT}{(2r_b + r_e + 2R_2)\left|\frac{Z_G}{R_1}+1\right|^2 + 2R_1\left|\frac{Z_G}{R_1}\right|^2 + \left[\frac{(r_e + r_b + R_2)\left|Z_G/R_1 + 1\right| + \left|Z_G\right|}{\beta r_e}\right]^2} \tag{20}$$

In a reproduce head, assuming a constant flux recording and disregarding the gap loss at high frequencies, the open circuit output voltage of the head is proportional to the frequency and the number of turns of wire on the head.

$$E_H \propto f N \tag{21}$$

The inductance of the head is proportional to the number of turns squared.

$$L_H \propto N^2 \tag{22}$$

Combining equations (21) and (22) and solving for the reproduce head voltage at any frequency:

$$(E_H)^2 = Cf^2 L_H \tag{23}$$

where C is a proportionality constant depending upon the type of reproduce head. The equation for the unequalized signal-to-noise power at any single frequency f, and as with the resistive source, assuming $R_1 >> Z_G$ is then:

$$S/N = \frac{Cf^2 L_H/2KT}{2R_b + 2R_2 + R_e + \frac{(R_b + R_e + R_2)^2 + (2\pi f L_H)^2}{\beta R_e}} \tag{24}$$

In tape recorder reproduce amplifiers, the criteria of signal-to-noise is measured by the equalized output of the amplifier to the total noise of the reproduce system. The reproduce amplifier has the inverse frequency characteristic of the reproduce head (assuming a constant flux recording). Thus, the amplifier gain function squared becomes:

$$\left[G\left(\frac{f}{f_0}\right)\right]^2 = G^2 \frac{f_0^2}{f^2} \tag{25}$$

where G = gain at f_0 = f
f = any frequency
f_0 = reference frequency

The ratio of the equalized output signal to broadband noise between frequencies f_1 and f_2, is then:

$$S/N = \frac{CL_H/2KT}{\int_{f_1}^{f_2}\left[\frac{2R_b + R_e + 2R_2}{f^2} + \frac{(R_e + R_b + R_2)^2}{f^2 \beta R_e} + \frac{(2\pi L_H)^2}{\beta R_e}\right] df} \tag{26}$$

and performing the indicated integration becomes:

$$S/N = \frac{CL_H/2KT}{\left[2R_b + 2R_2 + R_e + \frac{(R_e + R_b + R_2)^2}{\beta R_e}\right]\left[\frac{1}{f_1} - \frac{1}{f_2}\right] + \frac{(2\pi L_H)^2 (f_2 - f_1)}{\beta R_e}} \tag{27}$$

OPTIMUM SOURCE INDUCTANCE

Equation (27) is interesting in that it contains L_H in the numerator and L_H^2 in the denominator and thus should possess some value of L_H for which the signal-to-noise ratio is a maximum. Differentiating S/N to L_H and setting the resultant to zero gives the optimum head inductance as:

$$L_H{}^2{}_{OPT.} = \frac{\beta r_e (2 r_b + 2R_2 + r_e) + (r_e + r_b + R_2)^2}{(2\pi)^2 f_1 f_2} \tag{28}$$

If L_H opt. is put into equation (27), the S/N ratio is:

$$S/N_{MAX.} = \frac{C \beta r_e \sqrt{f_1 f_2}}{8\pi KT(f_2 - f_1)\sqrt{\beta r_e (2 r_b + 2R_2 + r_e) + (r_e + r_b + R_2)^2}} \tag{29}$$

The important point to notice about equation (29) is that a fractional expansion of the equation contains only re in the numerator, thus implying that with the higher re (the lower I_C), the better the signal-to-noise ratio. As with the resistive source R_G, a limit in the decrease of noise is reached when the beta (β) of the transistor decreases at the lower transistor current levels.

EXPERIMENTAL VERIFICATION

To verify the general conclusions, an equalized transistor amplifier was constructed with provision for varying the source inductance and the current through the first stage transistor. The inductances were identical types of reproduce heads, with only the number of turns on each head being different.

The output of the amplifier was connected to an eighteen db per octave cutoff filter at 1 KC and 15 KC. The lower limit of 1 KC was chosen to avoid 1/f noise of the transistor and to eliminate 60 cps and its harmonics from introducing an error in the noise measurements. The bias resistor R_1 was kept much greater than R_G and the emitter to collector voltage of the transistor was maintained at five volts.

Figures 5, 6 and 7 are plots of the measured and calculated S/N values for emitter currents of 1 ma, 300μa, and 100μa respectively. For these measurements, R_2 was 430 ohms. The measured and calculated values show a close correlation. The calculated S/N ratio in all three figures shows a slightly higher S/N ratio than does the measured values. The probable reason for this was the noise sources in the reproduce heads which was not taken into account. The curves clearly show that a maximum S/N ratio exists for a transistor amplifier operating with an inductive source, such as a tape recorder reproduce head. Additionally, the maximum S/N ratio increases with a decrease in bias current.

CONCLUSION

For inductive sources such as tape recorder heads feeding into an equalized amplifier, there exists an optimum source inductance for any current level in the transistor, which will result in the highest signal-to-noise power ratio. If an optimum inductance can be used at every current level of the transistor, then the signal-to-noise ratio will increase as the current through the transistor decreases, limited finally by the decrease in current gain of the transistor at lower current levels.

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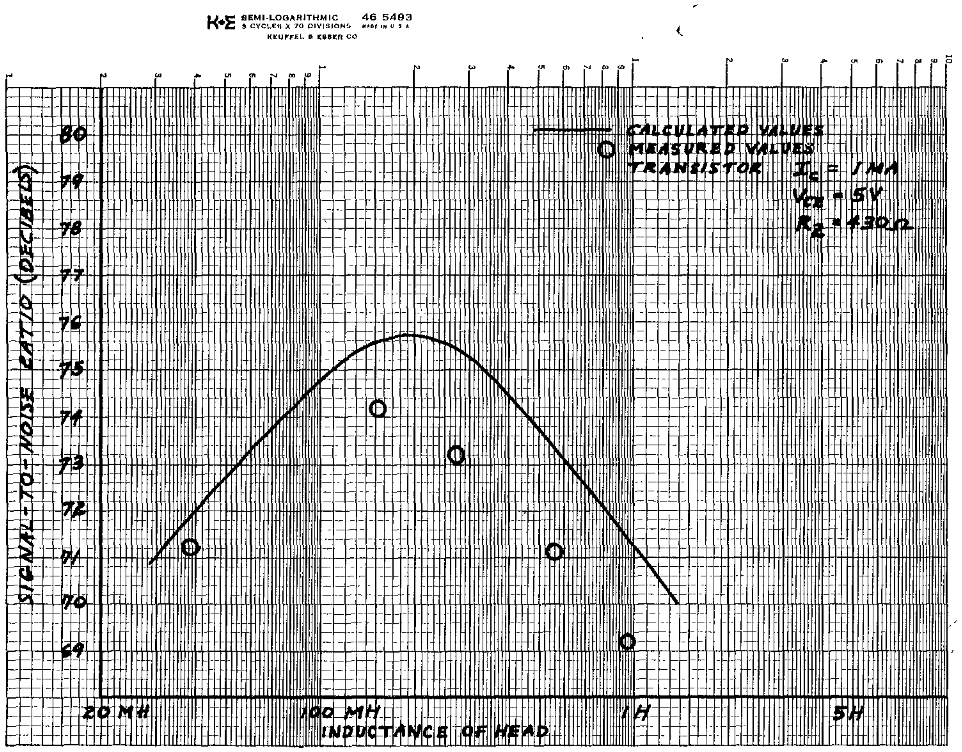

FIGURE 5

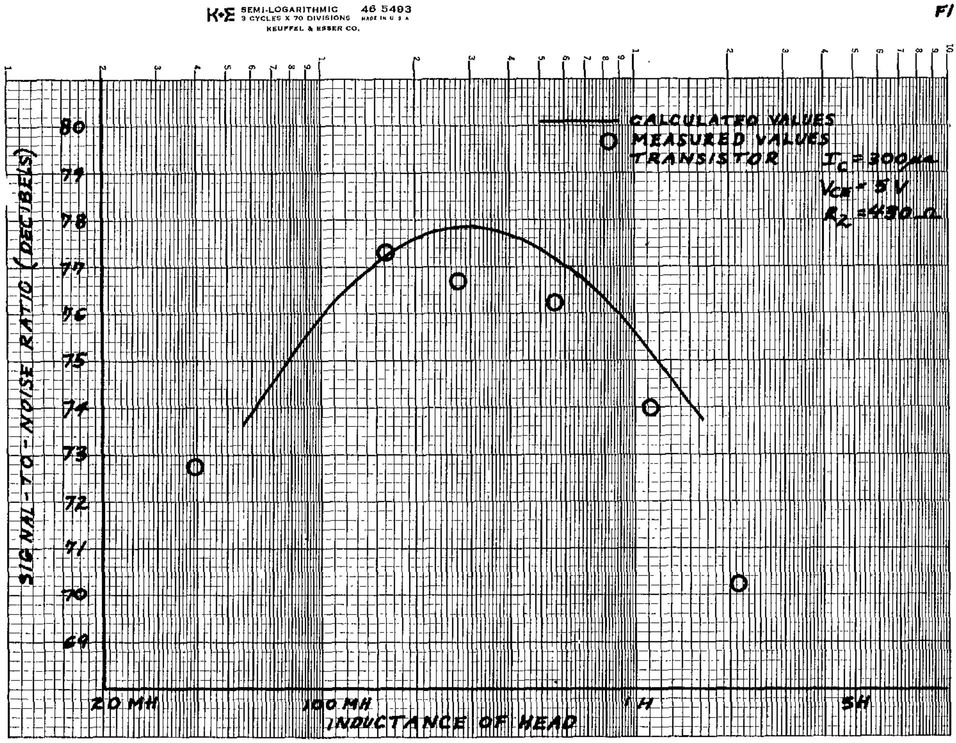

FIGURE 6

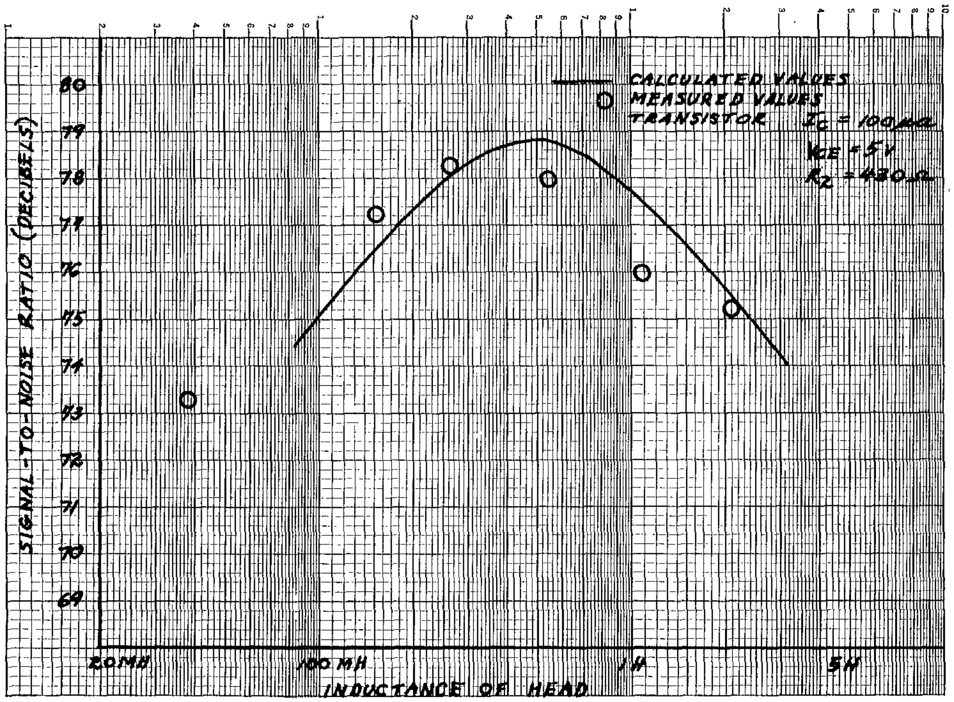

FIGURE 7