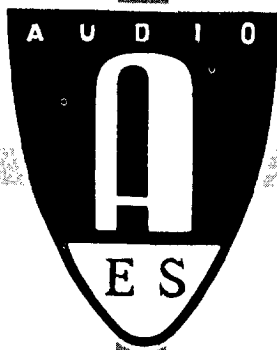


SINGLE ENDED ELECTROSTATIC LOUDSPEAKERS WITH  
HIGH EFFICIENCY AND IMPROVED LINEARITY

by  
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# SINGLE ENDED ELECTROSTATIC LOUDSPEAKERS WITH HIGH EFFICIENCY AND IMPROVED LINEARITY

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An electrical circuit analogue is developed to give the electrical input-acoustical output characteristics for electrostatic loudspeakers. A single ended wide range tweeter, having one watt output to 200 cps, is designed with the model and has an electro-acoustic feedback loop and tapered diaphragm supports. Tests with a pink noise source and a spectrum analyzer are described.

## Introduction

Electrostatic transducers and their basic operating principles have been known for many years and recently have been associated with frequency response linearity and good transient handling capability. These desirable characteristics have been possible because of the availability of thin diaphragm materials which have negligible mass loading in the audible range. With a simple flat disk type push pull electrostatic loudspeaker, the diaphragm forces and motion are nearly constant over a great percentage of the total area. Standing waves, diaphragm breakup and similar problems encountered in wide range stiff piston designs are eliminated and the frequency response, though possibly not flat, is free of sharp irregularities which give coloration that is almost impossible to equalize.

Unfortunately the average narrow band power density of music increases about 3 db/octave below a few hundred cycles and the peaks at these frequencies are apt to be much greater in comparison to the average than at higher frequencies. Since the diaphragm excursion for constant output power increases with lower frequencies, the distortion from compliance and electrical force nonlinearities is aggravated by the requirement for increased low frequency output. A large diaphragm with electrodes spaced a larger distance from it may be used; However, large polarizing voltages, low sensativity, poor amplifier matching, and directional high frequencies result from the design compromise.

If good subjective performance is an important requirement then a listener is aware of only the poor directionality and not of the other problems associated with the hypothetical design. His requirements would be met by a cylindrical segment configuration whose height and curvature would allow him to be normal to any part of the diaphragm. This transducer, which gives an equivalent finite line sound source, is not a very practical push pull design because of strict mechanical tolerances and rigidity requirements to maintain a uniform electrode to diaphragm spacing. Construction difficulties are further aggravated if a spherical segment type geometry is considered.

Several designs have evolved to help eliminate these problems and to yield a device which is practical to construct. The full range electrostatic loudspeakers generally employ two or more restricted range transducers, each with a minimum of overall design compromise. In most cases, the low frequencies are handled by flat plate push pull devices which cross over at fairly high frequencies to a curved diaphragm tweeter. The necessary crossover networks have subtle construction to equalize response non-linearities but have not been successful as far as amplifier matching is concerned. A full range design seen occasionally in Europe uses narrow full range elements cylindrically clustered to form a rather large affair which doesn't have crossover problems. A third contrasting design is the restricted range cylindrical segment high frequency tweeter of single ended construction. This design has a resonant peak type of response which is usually equalized by the crossover network. It is simple to construct, but it can give very high nonlinear distortion if allowed to operate over the range of push pull devices with similar dimensions.

A single ended electrostatic tweeter design, to be described, eliminates many of the previously mentioned problems. Though not of full range capability, its very low cutoff frequency makes it adaptable to polystyrene piston type woofers for use in an extremely high quality system. The distortion from nonlinear field and compliance effects expected from the single ended design are decreased by more than 20 db. by a negative feedback electrode. In addition a tapered diaphragm support distributes the natural resonant frequencies over a fairly broad range so that RC type equalization may be employed prior to the output stage. With this design an average output of well over one acoustic watt is possible to 200cps with standard vacuum tube amplifiers and polarizing voltages less than six hundred volts.

## An Electro-acoustical Analogue

A practical electrostatic loudspeaker will have its parameters optimized for subjective performance and for presenting the driving amplifier with an acceptable load which doesn't require a great range of current and voltage. In addition the feedback loop required for successful single ended operation must be stable and consequently the diaphragm motional phase versus applied voltage response must have less than  $180^\circ$  phase shift over the desired passband.

A model giving the transfer characteristics of the diaphragm motion versus the applied voltage will be developed and from this model the efficiency, input impedance, and phase response can be found. A capacitor with a single plate area  $A$ , spacing  $S_p$  and having an applied voltage  $V_p$  is considered similar to a transducer with the same diaphragm area. The energy stored in watt-seconds is:

$$W_c = \frac{1}{2} C V_p^2$$

$$\text{Where } C = \frac{A\epsilon}{S_p}$$

and  $\epsilon =$  permittivity of space

since force  $\Rightarrow \frac{dW}{dS}$  then

$$F = - \frac{A\epsilon V_p^2}{2 S_p^2}$$

The differential force for a small alternating voltage is

$$\frac{dF_p}{dV_p} = - \frac{A\epsilon V_p}{S_p^2}$$

$$\text{or } f(t) = \frac{A\epsilon V_p}{S_p^2} v(t)$$

Where  $f(t)$  = the peak alternating force

$v(t)$  = the peak alternating voltage

A back generated voltage will be developed between the capacitor terminals when the plates move with respect to each other.

$$\text{if } Q = C V_p = \text{a constant}$$

$Q$  = stored charge

$$\text{then } \frac{dV_p}{dS_p} = \frac{Q}{A\epsilon} = \frac{V_p}{S_p}$$

$$\text{or } v(t) = \frac{V_p}{S_p} \omega(t)$$

where  $\omega(t)$  = the peak alternating displacement between plates

The terminal voltage between the plates is the sum of the voltages from the charging current and the motional back emf.

$$V(t) = V_c(t) + V_B(t) = \int \frac{1}{C(t)} i(t) dt + \frac{V_p}{S_p} \omega(t)$$

where  $V_c(t)$  = voltage across an equivalent charged capacitor

$V_f(t)$  = motional back generated voltage

Small displacements and signals are assumed so that the  $1/C$  term in the integral is nearly constant during a suitable length of time. An equivalent circuit of a series capacitor and voltage generator is then valid. In order to determine the electrical impedance of this network, the mechanical diaphragm loading must be known. This load impedance is composed of four significant effects and is approximately determined by considering the diaphragm as a movable piston operating in an infinite baffle. Such an assumption is valid in most single ended designs because the actual electrical crossover frequency is above the natural acoustic cutoff frequency where the diaphragm to air coupling becomes poor. Consequently the largest reproduced wavelength will seldom be larger than the smallest diaphragm dimension and acoustic load,  $Z_m(\omega)$ , will be practically real and constant.

$$Z_q = \text{acoustic diaphragm load} = \frac{A'}{\rho C}$$

where

$$\lambda < R$$

$A'$  = area of an equivalent piston =  $\text{cm}^2$

$\rho$  = density of air

$C$  = speed of sound

$$Z_m \cong 42 A = \text{mechanical } \Omega$$

Additional loads occur from slot loading between the diaphragm and solid portions of the perforated electrode and compliant effects from the stretched diaphragm materials. Normally the electrode perforations will each be large enough to present negligible losses. Such is not true with the solid portions which, with small  $s_p$ , form very narrow slots from which the back radiation must pass. This loading is approximately that of many small pistons each spaced from a solid material by  $s_p$  and each of a diameter slightly greater than the shortest distance between perforations. The impedance in mechanical ohms is:

$$Z_s(\omega) = \frac{.022 r^2 \eta A}{s_p^2} - j \frac{.009 r^4 \eta A f}{s_p}$$

where

$s_p$  = spacing in cm.

$r$  = radius to the nearest perforation edge as measured from the centroid of the smallest perforation cluster

$n$  = the number of symmetrical clusters per cm.<sup>2</sup>

$A$  = the diaphragm area

$f$  = frequency in cps.

Compliant loading from the diaphragm material increases as the cell dimensions are reduced. However, if the material yield strength is improved the cell dimensions can be made larger without danger of the diaphragm collapsing to the electrode from the field pressure. For a practical device the cell width is a function of polarizing voltage, yield strength, and predicted material creep and is best determined by experiment. The spring constant for this cell dimension is:

$$K = \frac{d}{4 l F_T} = \frac{\text{cm.}}{d_y n e} / \text{cm.}$$

The compliance loading is then

$$X_c = \frac{l F_T}{f d 2\pi}$$

$l$  = cell length

$F_T$  = dynes/cm cell length

$f$  = frequency in cps.

$d$  = cell width

When the diaphragm loading effects are combined an electro-mechanical model and its differential equations is formulated. In addition the back generated voltage, developed when the diaphragm moves, can be written directly and is very useful for solving the acoustic output with voltage or current inputs and for making s plane plots for feedback stability analysis. This back voltage in terms of frequency is;

$$V_B(f) = V_a(f) \frac{A \epsilon V_p}{S_p} \left[ \frac{1}{\frac{A}{\epsilon C} + \frac{.022 r^2 \eta A}{S_p^2} - j \left( \frac{.009 \eta A f}{S_p} - \frac{\lambda F_T}{2\pi f d} \right)} \right]$$

The input current to the equivalent series input capacitor is

$$i(\omega) = V_c(\omega) j\omega C$$

$$\begin{aligned} Z_{in} &= \frac{V_a(f)}{V_a(f) - [V_B(f)] j 2\pi f C} \\ &= \frac{1}{j 2\pi f C \left[ 1 - \frac{A'}{B + j 2\pi f C' + \frac{D}{j 2\pi f}} \right]} \end{aligned}$$

where

$$A' = \frac{A \epsilon V_p}{S_p^2}$$

$$B = \frac{A}{\epsilon C}$$

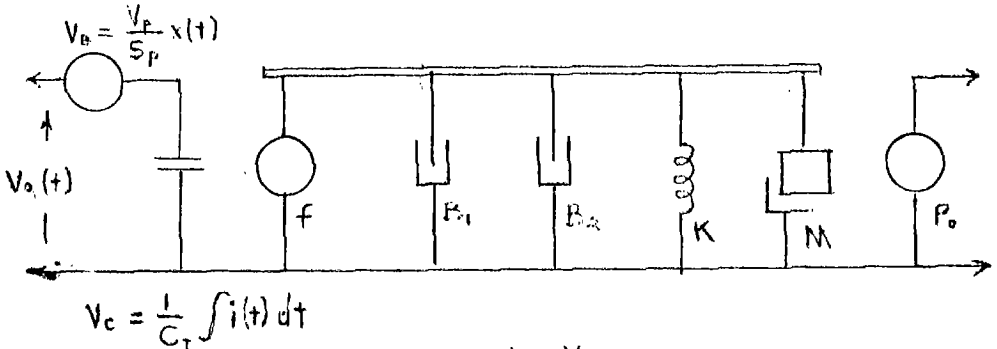
$$C' = \frac{.022 r^2 \eta A}{S_p^2}$$

$$D = \frac{.009 r^4 A f}{S_p}$$

$$E = \frac{\lambda F_T}{2\pi f d}$$

$$\text{then } Z(s) = \frac{C's^2 + Bs + D}{s(CC's^2 + (B-A)Cs + DC)}$$

The electromechanical model obeying these relations is developed from the physical system by inspection, and upon modifying its differential equations is converted into an electrical circuit describing the model.



$$f = \frac{A \epsilon V_p}{S_p} v(t)$$

$$K = \frac{d}{\lambda F_T}$$

$$B_1 = 42 A$$

$$B_2 = \frac{.022 r^2 \eta A}{S_p^2}$$

$$M = \frac{.009 r^4 \eta A}{2 \pi S_p}$$

$$P_o = .764 \times 10^{-10} (Df)^4 \left(\frac{S_p}{V_p}\right)^2 v_b^2$$

= acoustic watts output rms from one side of the diaphragm

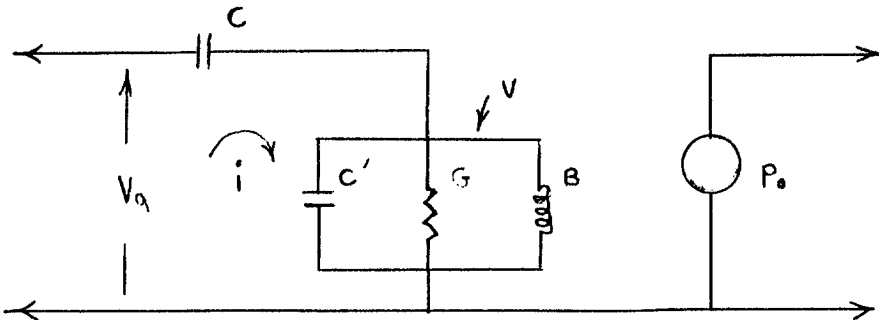
From Kirchhoff's and Newton's laws, the differential equations for this one loop electromechanical system are written and are modified to resemble the form associated with electrical networks. These equations and the corresponding circuit given below form a model which will closely approximate the physical system.

$$C \frac{u}{V} \left[ M \frac{d^2 V}{dt^2} + (B_1 + B_2) \frac{dV}{dt} + \left( K - \frac{V}{u} \right) V \right] = \int i dt$$

$$\frac{1}{C} \int i dt = V_a - V$$

$$M = \frac{A \epsilon V_p}{S_p} = \text{force constant}$$

$$V = \frac{V_p}{S_p} = \text{back voltage constant}$$



$$C = \frac{A \epsilon}{S_p}$$

$$C' = C \frac{u}{V} M = \left[ \frac{A r^4 \eta}{V_p^2} \times \frac{0.009}{2\pi} \right]$$

$$G = \frac{S_p}{V_p^2} \left( 42A + \frac{0.022 r^2 \eta A}{S_p^2} \right)$$

$$B = \frac{S_p^4}{V_p^2} \left( \frac{d}{l F_T} - \frac{V_p^2}{S_p} \right)$$

This model in the circuit or differential equation form is easily solved on an analog computer for the driving point impedance, and the transfer characteristics as a function of important parameters. Of greatest concern is the behavior when  $S_p$  is varied since this parameter appears in most terms as a high order effect and determines the output linearity to be expected from a given diaphragm displacement. If the diaphragm yield strength and electric field strength are constant when  $S_p$  is reduced, the resonant frequency will at first decrease and then as  $S_p$  becomes smaller will increase very rapidly. The mechanical resonant  $Q$  is continually decreasing and the ratio of acoustic output to slot dissipation becomes less. In addition, the resistive component of the input impedance becomes greater and consequently the acoustic output, with a voltage driving source, becomes linear over a greater fraction of the useful bandwidth. This drop in  $Q$  allows better phase stability when an active feedback loop is used to improve the amplitude linearity. Even though the open loop distortion for a given diaphragm displacement increases with smaller  $S_p$ , the greater permissible feedback will tend to compensate for some of the distortion increase. The end result is that a single ended "close spaced" electrostatic transducer having low polarizing and driving voltage requirements can reproduce levels comparable to designs requiring impractical voltages and amplifiers.

#### Construction and Test of an Extended Range Tweeter

A single ended electrostatic loudspeaker with its perforated stationary electrode made from standard 4 ft. by 5 ft. sheet stock is designed for a 200cps. crossover and uses a 0.00025 inch metalized mylar diaphragm. The angular dispersion, which is a function of the segment angle, is  $135^\circ$  and the maximum polarizing voltage is 600 volts. For a one watt rms. output with 20% field distortion, a diaphragm spacing of 0.0045 average is required. The cell width is then experimentally measured with the full peak voltage as about 0.7 inches. These parameters are applied to the analogue and the input impedance and amplitude linearity are solved and evaluated for the particular amplifier and subjective requirements desired. If the solution is undesirable the area and spacing parameters are modified and the procedure is repeated.

The feedback electrode, used for reducing distortion, is a narrow shielded strap of perforated metal placed vertically and about 0.020 inches from the diaphragm and contains an internal cathode follower amplifier. The shielding is essential since the diaphragm to electrode capacity is very low thereby making the feedback path sensitive to external noise.

Supporting the diaphragm by unequally spaced nylon cords distributes the natural frequencies to broaden the midband resonant peaks. With these tapered supports it is possible to achieve very flat frequency response with with simple RC type equalizers.

Performance tests are made using white or "pink" random noise sources with restricted bandwidth to excite the driving amplifier and tweeter. These measurements are taken outdoors and the output from a carefully placed condenser microphone is set at a calibrated level and recorded on magnetic tape. This recording is reproduced at an increased speed into a spectrum analyzer to present a direct plot of the amplitude response. The test arrangement, fig. 3 and 4, eliminates the cavity and space standing wave problems encountered with steady state measurements in rooms with reflecting surfaces. In addition the test speakers are subjected to waveforms more closely resembling those of music. Multiplying the tape speed multiplies the spectrum frequencies thereby giving a more rapid narrow band analysis than is possible in real time. Random noise measurements on electrostatic and cone type speakers is shown in figs. 4,5,6,7. For these plots, the rms. microphone output voltage and its bandwidth are such that the average output from one side of the diaphragm is about  $\frac{1}{4}$  watt for each of the figures. Figs. 4 and 5 show the white noise input voltage vs. current for small diaphragm spacings in otherwise identical tweeters. Although both power inputs are about the same, the tweeter with the larger  $S_p$  requires an amplifier of greater output capability to supply the extra circulating current. Figs. 6 and 7 show the frequency response of the cone and electrostatic tweeters as plotted with a spectrum analyzer. The cone speaker is a high efficiency extended range type which is 8" in diameter and is mounted unsymmetrically on a heavy board. The severe peaks of the cone device are nonlinear effects which rapidly subside as the driving power is reduced.

Although high distortion and somewhat rough frequency response are usually the rule with single ended designs, the addition of a feedback loop and tapered diaphragm supports can substantially decrease these effects if the system to which they are added is carefully designed. For large extended range tweeters, the extra expense of these additions is much less than that for the larger amplifiers stricter mechanical tolerances and higher voltages needed for push pull devices of similar performance.

- 1 "Acoustic Design Charts", Frank Massa, The Blakiston Co.  
Philadelphia, 1942
- 2 "Transients in Linear Systems", M.F. Gardner and J. L. Barnes  
Vol. I, John Wiley and Sons, N.Y., 1942
- 3 "Elements of Acoustical Engineering", H. F. Olson,  
D. Van Nostrand Co., N.Y., 1940

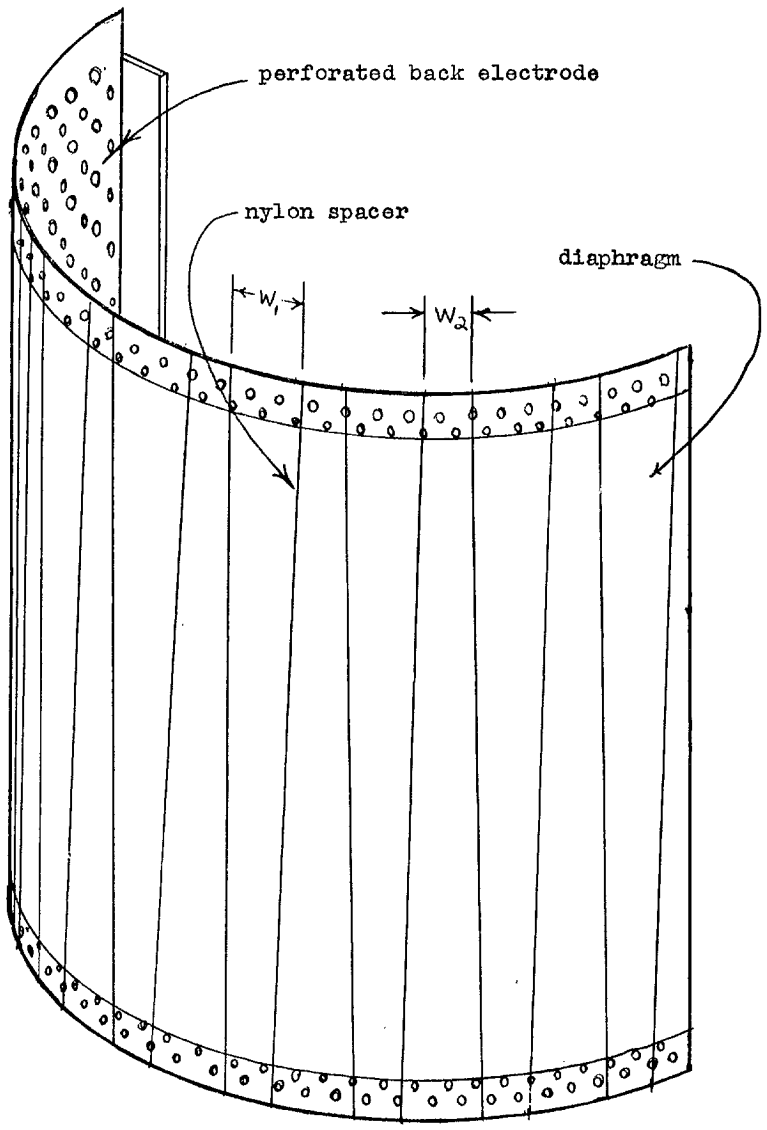


Fig. 1 A single ended electrostatic tweeter with tapered diaphragm supports and a cylindrical segment shape

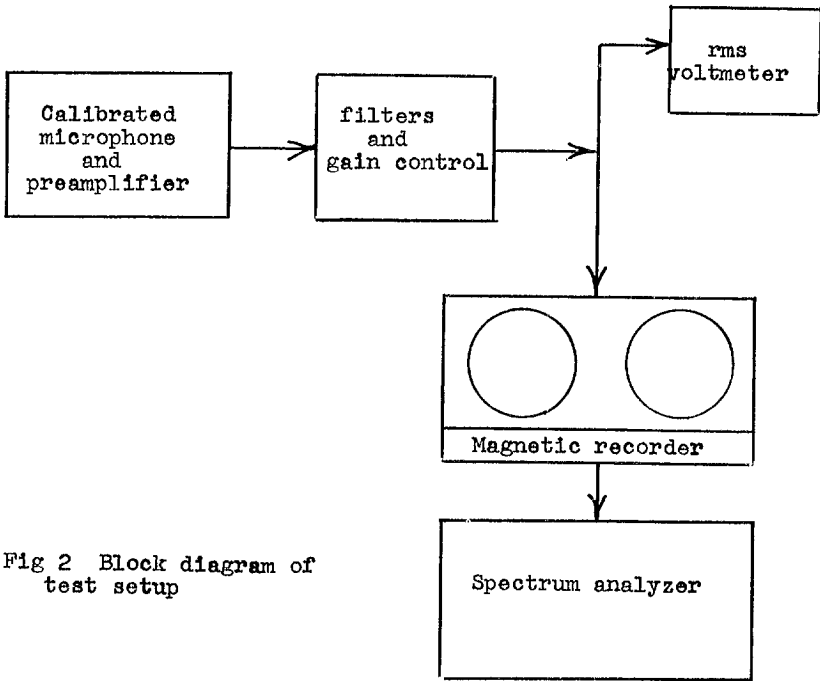
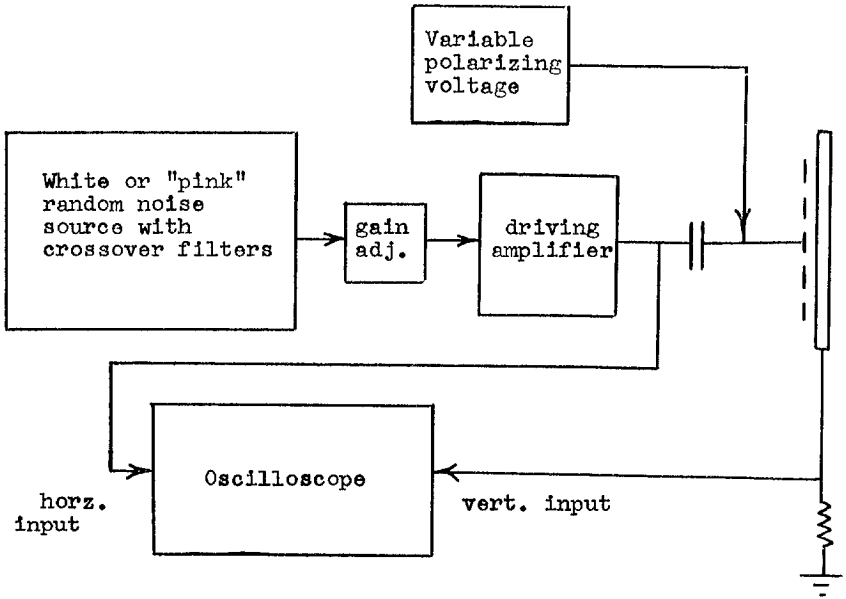


Fig 2 Block diagram of test setup

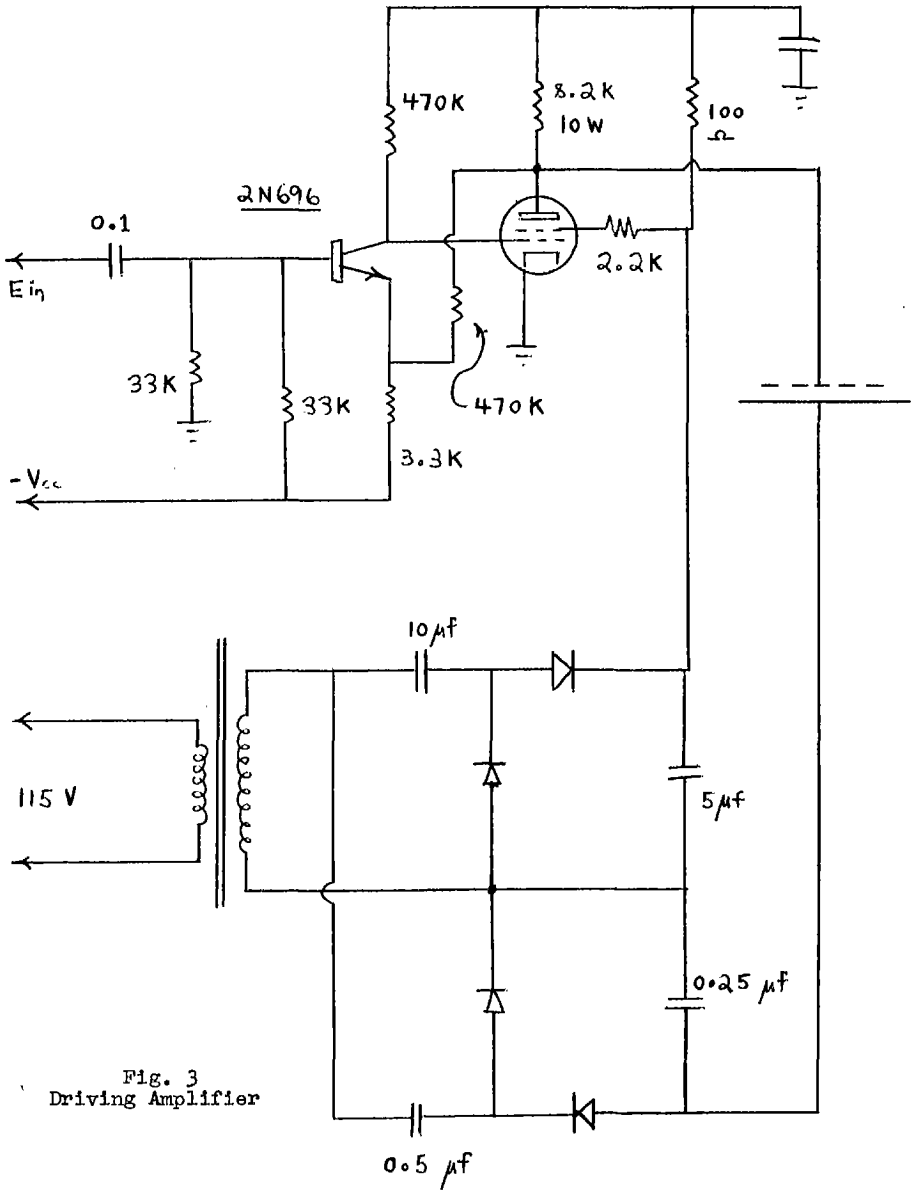


Fig. 3  
Driving Amplifier

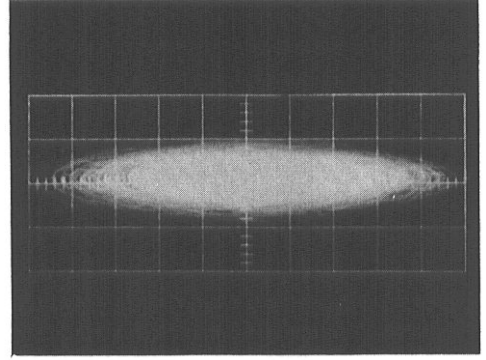
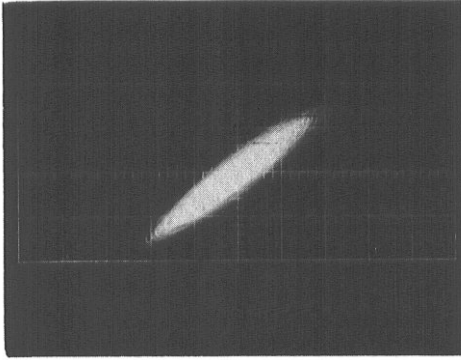


Fig. 4 ( $S_p$  0.0045 average)      Fig. 5 ( $S_p$  0.010 av.)  
 White noise voltage vs. current at input terminals.  
 The abscissa is volts times 100 and the ordinate  
 is current in tenths of an amp.

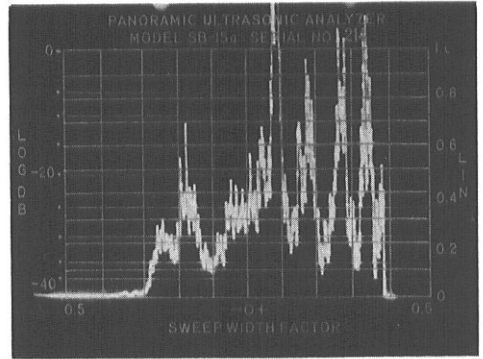
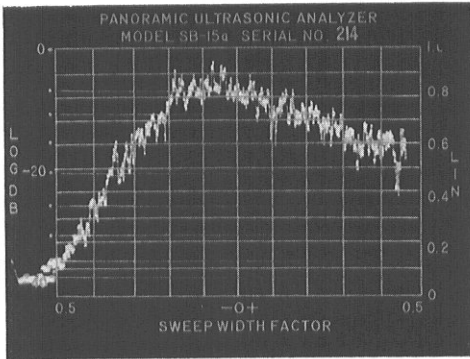


Fig. 6 Electrostatic tweeter      Fig. 7 cone type speaker  
 White noise spectrum plots  
 Vertical is in db volts from microphone  
 Horizontal is frequency in cps. linearly increasing  
 from left to right at 1.5 kc./division